Pearson Edexcel

Mark Scheme (Result)

October 2020

Pearson Edexcel GCE In A level Further Mathematics
Paper 9FM0/3A

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- M marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- A marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- B marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- □ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

Question	Scheme	Marks	AOs
1	$\dfrac{\frac{\mathrm{d}}{\mathrm{d}x}\left(\mathrm{e}^{\sin x}-\cos(3x)-\mathrm{e}\right)}{\frac{\mathrm{d}}{\mathrm{d}x}(\tan(2x))}=\dfrac{\pm\cos(x)\mathrm{e}^{\sin x}\pm A\sin(3x)}{B\sec^2 2x}$	**M1**	1.1b
	$\dfrac{\frac{\mathrm{d}}{\mathrm{d}x}\left(\mathrm{e}^{\sin x}-\cos(3x)-\mathrm{e}\right)}{\frac{\mathrm{d}}{\mathrm{d}x}(\tan(2x))}=\dfrac{\cos(x)\mathrm{e}^{\sin x}+3\sin(3x)}{2\sec^2 2x}$	**A1** **A1**	1.1b 1.1b
	$\lim\limits_{x\to\frac{\pi}{2}}\dfrac{\cos(x)\mathrm{e}^{\sin x}+3\sin(3x)}{2\sec^2 2x}$ $=\dfrac{\cos\left(\frac{\pi}{2}\right)\mathrm{e}^{\sin\left(\frac{\pi}{2}\right)}+3\sin\left(\frac{3\pi}{2}\right)}{2\sec^2\left(\frac{2\pi}{2}\right)}$ or $=\dfrac{0\times\mathrm{e}+3\times(-1)}{2\times(-1)^2}=\ldots$	**M1**	1.2
	$=-\dfrac{3}{2}$ *	**A1***	2.1
		(5)	
		(5 marks)	

Notes:

M1: Attempts differentiation of both numerator and denominator, including at least one use of the chain rule. Either numerator or denominator of the correct form. May be done separately.

A1: Numerator correct

A1: Denominator correct

M1: Applies l'Hospital's Rule, must **see clear use of a substitution** of $x=\frac{p}{2}$ into their derivatives, not the original expression. (no need to see check that limits of numerator and denominator are non-zero).

A1*: Needs to be a correct intermediate line following substitution before reaching the printed answer with use of some limit notation. All aspects of the proof should be clear for this mark to be awarded and no errors seen.

Question	Scheme	Marks	AOs
2	Step $\frac{1}{3}$	B1	1.1b
	<table><tr><td></td><td>y_0</td><td>y_1</td><td>y_2</td><td>y_3</td><td>y_4</td><td>y_5</td><td>y_6</td></tr><tr><td>x</td><td>-1</td><td>$-\frac{2}{3}$</td><td>$-\frac{1}{3}$</td><td>0</td><td>$\frac{1}{3}$</td><td>$\frac{2}{3}$</td><td>1</td></tr><tr><td>y</td><td>0</td><td>2.2981</td><td>2.9544</td><td>3</td><td>2.9544</td><td>2.2981</td><td>0</td></tr></table>	M1	3.4
	$y_0+4y_1+2y_2+4y_3+2y_4+4y_5+y_6=\text{"42.203"}$ $\{0+4(2.2981+3+2.2981)+2(2.9544+2.9544)+0\}$	M1	1.1b
	$=42.203\left(=24\cos\left(\frac{2\pi}{9}\right)+12\cos\left(\frac{\pi}{18}\right)+12\right)$	A1	1.1b
	So volume required is approx. $\frac{85}{1000}\times\frac{\frac{1}{3}}{3}\times\text{"42.203"}$	M1	3.1a
	= awrt 0.3986 m^3	A1	3.2a
	Alternative interval [0,1]) step $\frac{1}{6}$ and the answer is doubled later	B1	1.1b
	<table><tr><td></td><td>y_0</td><td>y_1</td><td>y_2</td><td>y_3</td><td>y_4</td><td>y_5</td><td>y_6</td></tr><tr><td>x</td><td>0</td><td>$\frac{1}{6}$</td><td>$\frac{1}{3}$</td><td>$\frac{1}{2}$</td><td>$\frac{2}{3}$</td><td>$\frac{5}{6}$</td><td>1</td></tr><tr><td>y</td><td>3</td><td>2.9971</td><td>2.9544</td><td>2.7716</td><td>2.2981</td><td>1.3852</td><td>0</td></tr></table>	M1	3.4
	$y_0+4y_1+2y_2+4y_3+2y_4+4y_5+y_6=\text{"42.1206"}$ $\{3+4(2.9971+2.7716+1.3852)+2(2.9544+2.2981)+0\}$	M1	1.1b
	Awrt 42.121	A1	1.1b
	So volume required is approx. $\frac{85}{1000}\times\frac{\frac{1}{6}}{3}\times\text{"42.1206"}\times 2$	M1	3.1a
	= awrt 0.3978 m^3	A1	3.2a
		(6)	
			(6 marks)

Notes:

B1: Correct strip width for the method chosen $\frac{1}{3}$ for the interval $[-1,1]$

M1: Uses the model to find the appropriate values for the method. May use that the function is even to only work out half of them, so may be implied by use in the formula. At least two correct values to 4 s.f. needed for the method.

M1: Applies the "bracket" of Simpson's rule, "$y_0+4y_1+2y_2+4y_3+2y_4+4y_5+y_6$". Coefficients must be correct.

A1: Correct value for the "bracket". If not explicitly seen, may be implied by awrt 4.689 as a value for the cross section area following correct values.

M1: Correct full method to find the volume. E.g. multiplies their bracket by their $\frac{h}{3}$ and by 0.085. Accept an attempt in any consistent units, so e.g. in mm^3 ie $85\times\frac{\frac{1}{3}}{3}\times"42.203"\times1000^2$ **A1:** Correct answer in m^3.
B1: Correct strip width for the method chosen $\frac{1}{6}$ for the interval [0,1] and later doubled. **M1:** Uses the model to find the appropriate values for the method. May use that the function is even to only work out half of them, so may be implied by use in the formula. At least two correct values to 4 s.f. needed for the method. **M1:** Applies the "bracket" of Simpson's rule, $"y_0+4y_1+2y_2+4y_3+2y_4+4y_5+y_6"$. Coefficients must be correct. **A1:** Correct value for the "bracket". If not explicitly seen, may be implied by awrt 4.680 as a value for the cross section area following correct values. **M1:** Correct full method to find the volume. E.g. multiplies their bracket by their $\frac{h}{3}$ and by 0.085. Accept an attempt in any consistent units, so e.g. in mm^3 ie $85\times\frac{\frac{1}{6}}{3}\times"42.203"\times1000^2\times2$ **A1:** Correct answer in m^3.

<u>Using 6 ordinates</u>

Max score B0 M1 M0 A0 M0 A0

	y_0	y_1	y_2	y_3	y_4	y_5
x	-1	-0.6	-0.2	0.2	0.6	1
y	0	2.53298	2.9941	2.9941	2.53298	0

B0: Incorrect strip width

M1: Uses the model to find the appropriate values for the method. At least two correct values to 4 s.f. needed for the method.

Question	Scheme	Marks	AOs
3(a)	$\overrightarrow{AB} = -2\mathbf{i}+6\mathbf{j}+4\mathbf{k}$ and $\overrightarrow{AC} = -5\mathbf{i}+5\mathbf{j}+2\mathbf{k}$	**B1**	1.1b
	$\overrightarrow{AB}\times\overrightarrow{AC} = \begin{vmatrix} -2 & 6 & 4 \\ -5 & 5 & 2 \end{vmatrix}$ $= (6\times2-4\times5)\mathbf{i}-(-2\times2-4\times-5)\mathbf{j}+(-2\times5-6\times-5)\mathbf{k}$	**M1**	1.1b
	$= -8\mathbf{i}-16\mathbf{j}+20\mathbf{k}$	**A1**	1.1b
		(3)	
(b)	E.g. $\mathbf{n} = -8\mathbf{i}-16\mathbf{j}+20\mathbf{k}$ gives $p = (3\mathbf{i}-2\mathbf{j}+\mathbf{k}).(-8\mathbf{i}-16\mathbf{j}+20\mathbf{k}) = ...$ E.g. $\mathbf{n} = 2\mathbf{i}+4\mathbf{j}-5\mathbf{k}$ gives $p = (3\mathbf{i}-2\mathbf{j}+\mathbf{k}).(2\mathbf{i}+4\mathbf{j}-5\mathbf{k}) = ...$	**M1**	1.1b
	Equation is $\mathbf{r}.(-8\mathbf{i}-16\mathbf{j}+20\mathbf{k}) = 28$ or $\mathbf{r}.(2\mathbf{i}+4\mathbf{j}-5\mathbf{k}) = -7$ (oe)	**A1**	2.5
		(2)	
(c)	$\overrightarrow{AD}.\left(\overrightarrow{AB}\times\overrightarrow{AC}\right) = "\overrightarrow{AD}".(-8\mathbf{i}-16\mathbf{j}+20\mathbf{k}) = ...$	**M1**	1.1b
	$\overrightarrow{AD} = 5\mathbf{i}+9\mathbf{j}+4\mathbf{k}$	**B1**	1.1b
	Volume = $\frac{1}{6}\left\|\overrightarrow{AD}.(\overrightarrow{AB}\times\overrightarrow{AC})\right\| = \frac{1}{6}\left\|(5\mathbf{i}+9\mathbf{j}+4\mathbf{k}).(-8\mathbf{i}-16\mathbf{j}+20\mathbf{k})\right\| = ...$	**M1**	3.1a
	$= \frac{52}{3}$ o.e. $17\frac{1}{3}$	**A1**	1.1b
		(4)	
			(9 marks)

Notes:

(a)

B1: Both $\overrightarrow{AB}$ and $\overrightarrow{AC}$ correct.

M1: Applies the cross product to their $\overrightarrow{AB}$ and their $\overrightarrow{AC}$. There must be at least two correct components if no method seen. Method can be implied by $\mathbf{i}\begin{vmatrix} 6 & 4 \\ 5 & 2 \end{vmatrix} - \mathbf{j}\begin{vmatrix} -2 & 4 \\ -5 & 2 \end{vmatrix} + \mathbf{k}\begin{vmatrix} -2 & 6 \\ -5 & 5 \end{vmatrix} = ...$ with at least one correct component.

A1: Correct vector

(b)

M1: Uses their $\mathbf{n}$ (which may be any multiple of their $\overrightarrow{AB}\times\overrightarrow{AC}$) and any point on the plane in an attempt to find p. (Use of $\overrightarrow{AB}$ or $\overrightarrow{AC}$ is M0.)

A1: Correct equation in form stated. Accept any multiples, e.g. $\mathbf{r}.(-8\mathbf{i}-16\mathbf{j}+20\mathbf{k}) = 28$

(c)

M1: Attempts a suitable scalar triple product, e.g. $\overrightarrow{AD}.\left(\overrightarrow{AB}\times\overrightarrow{AC}\right)$. Must include a complete method to use all necessary vectors.

B1: Correct '$\overrightarrow{AD}$' if using $\overrightarrow{AD}.\left(\overrightarrow{AB}\times\overrightarrow{AC}\right)$, or all vectors correct if using a different product.

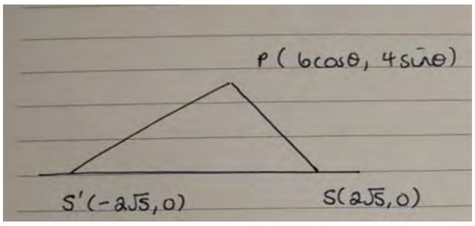
M1: Use of volume = $\frac{1}{6}\left|\text{their } \overrightarrow{AD}.\left(\overrightarrow{AB}\times\overrightarrow{AC}\right)\right|$ (oe full method to find the volume).

A1: Correct exact answer.

Question	Scheme	Marks	AOs
	$\mathrm{f}(x) = x^4 \sin(2x) \quad u = x^4 \quad v = \sin(2x)$		
4	$u' = 4x^3, u'' = 12x^2, u''' = 24x, u^{(4)} = 24$ (and $u^{(n)} = 0$ for $n > 4$)	M1	1.1b
	$v' = 2\cos(2x), v'' = -4\sin(2x), v''' = -8\cos(2x), v^{(4)} = 16\sin(2x),$ $v^{(5)} = 32\cos(2x), v^{(6)} = -64\sin(2x), v^{(7)} = -128\cos(2x),$ $v^{(8)} = 256\sin(2x),$	M1 A1 A1	3.1a 1.1b 1.1b
	$\mathrm{f}^{(8)}(x) = x^4 \times 256\sin(2x) + 8 \times 4x^3 \times -128\cos(2x)$ Thus $+\frac{8\times 7}{2} \times 12x^2 \times -64\sin(2x) + \frac{8\times 7\times 6}{6} \times 24x \times 32\cos(2x)$ $+\frac{8\times 7\times 6\times 5}{24} \times 24 \times 16\sin(2x)$ $\mathrm{f}^{(8)}(x) = x^4 \times 256\sin(2x) + 8 \times 4x^3 \times -128\cos(2x)$ $+28 \times 12x^2 \times -64\sin(2x) + 56 \times 24x \times 32\cos(2x)$ $+70 \times 24 \times 16\sin(2x)$	M1	2.1
	$\mathrm{f}^{(8)}(\pi) = 0 - 4096\pi^3 - 0 + 1344 \times 2^5\pi + 0 \ \left(= -4096\pi^3 + 43008\pi\right)$	M1	1.1b
	Coefficient is $\frac{\mathrm{f}^{(8)}(\pi)}{8!} = \frac{1344 \times 2^5\pi - 4096\pi^3}{8! \text{ or } \{40320\}}$	M1	2.2a
	$= \frac{336\pi - 32\pi^3}{315}$ (So $a = 336$ and $b = -32$)	A1	2.1
		(8)	
		(8 marks)	

Notes:

M1: Establishes the non-disappearing derivatives of x^4. Allow slips in coefficients, but powers must decrease.
M1: Identifies the relevant derivatives for sin(2x), up to the 8th derivative or establishes the correct pattern. Look for alternating between sin and cos. Condone use of x.
A1: Correct sizes for the coefficients, allow sign errors for this mark (may be due to incorrect signs when differentiating sin and cos) Must have angle 2x.
A1: All derivatives correctly established. (Note the sin terms may be omitted if the student has made clear they will disappear, but if present they must be correct).
M1: Applies Leibnitz's theorem to get the 8th derivative with their expressions. Binomial coefficients must be present.
M1: Evaluates their 8th derivative at π
M1: Uses Taylor series – divides their value for $\mathrm{f}^{(8)}(\pi)$ by 8!
A1: Simplifies to the correct answer.
<u>Note:</u> If do not use Leibnitz's theorem then maximum M0 M0 A0 A0 M0 M1 M1 A0

Question	Scheme	Marks	AOs
5(a)	$b^2 = a^2(1-e^2) \Rightarrow 16 = 36(1-e^2) \Rightarrow e = ...$	**M1**	1.1b
	$e^2 = \frac{20}{36}$ or $\frac{5}{9}$ or $e = \frac{\sqrt{5}}{3}$	**A1**	1.1b
	Foci are $\left(\pm 2\sqrt{5}, 0\right)$	**A1**	1.1b
		(3)	
(b)	Perimeter $= PS + PS' + SS'$ where $PS + PS' = e(PM + PM') = ...$	**M1**	3.1a
	$= e \times \frac{2a}{e} = ...$	**M1**	2.2a
	$... + 2 \times 2\sqrt{5}$	**B1ft**	1.1b
	$= 12 + 4\sqrt{5}$ hence perimeter is constant for any P on E.*	**A1***	2.1
	 Perimeter $= PS + PS' + SS'$ $PS = \sqrt{\left(2\sqrt{5} - 6\cos\theta\right)^2 + (4\sin\theta)^2} = ... \left\{6 - 2\sqrt{5}\cos\theta\right\}$ $PS' = \sqrt{\left(2\sqrt{5} + 6\cos\theta\right)^2 + (4\sin\theta)^2} = ... \left\{6 + 2\sqrt{5}\cos\theta\right\}$	**M1**	3.1a
	$PS + PS' = \left(6 - 2\sqrt{5}\cos\theta\right) + \left(6 + 2\sqrt{5}\cos\theta\right) = B$	**M1**	2.2a
	$... + 2 \times 2\sqrt{5}$	**B1ft**	1.1b
	$= 12 + 4\sqrt{5}$ hence perimeter is constant for any P on E.*	**A1***	2.1
		(4)	
			(7 marks)

Notes:

(a)

M1: Uses $b^2 = a^2(1-e^2)$ with $a = 6$ and $b = 4$ to find a value for e^2 or e.

A1: Correct value for e or e^2

A1: Correct foci

(b)

M1: Forms a complete strategy to find the perimeter using general P and applies the focus directrix property to the sides PS and PS'

M1: Deduces the length of the two sides adjacent to P is a constant

B1ft: Uses SS' is twice their *ae* from (a)
A1*: Finds the value and makes conclusion that perimeter is constant for any *P* on *E*
Alternative
M1: Forms a complete strategy to find the perimeter using general *P*. Finds the lengths of PS and PS' using Pythagoras theorem and the general coordinate $(6\cos\theta, 4\sin\theta)$
M1: Deduces the length of the two sides adjacent to *P* is a constant, using trig identities.
B1ft: Uses SS' is twice their *ae* from (a)
A1*: Finds the value and makes conclusion that perimeter is constant for any *P* on *E*

Note: Using the property that *PS* + *PS'* = 2a both method marks may be awarded as long as a reason is given e.g. definition/property of an ellipse

Question	Scheme	Marks	AOs
6(a)	Establishes need for $\lvert 5t-31\rvert > \lvert 3t^2-25t+8\rvert$ and attempts to find all C.V.'s to form the critical region, e.g. via a sketch.	**M1**	3.1a
	$5t-31=3t^2-25t+8 \Rightarrow 3t^2-30t+39=0 \Rightarrow t=...$ $t=5\pm 2\sqrt{3}$	**M1** **A1**	1.1b 3.4
	$-(5t-31)=3t^2-25t+8 \Rightarrow 3t^2-20t-23=0 \Rightarrow t=...$ $t=(-1),\dfrac{23}{3}$	**M1** **A1**	2.1 3.4
	Selects "insides" $(-1<)\alpha<t<\beta, \gamma<\mathrm{t}<\delta$ where $\alpha<\beta<\gamma<\delta$	**M1**	2.2a
	$(-1<)0 \leqslant t<5-2\sqrt{3}$ or $\dfrac{23}{3}<t<5+2\sqrt{3}$	**A1**	1.1b
	Both regions, $0 \leqslant t<5-2\sqrt{3}$ and $\dfrac{23}{3}<t<5+2\sqrt{3}$	**A1**	2.3
	Establishes need for $\lvert 5t-31\rvert > \lvert 3t^2-25t+8\rvert$ and attempts to find all C.V.'s to form the critical region, e.g. via a sketch.	**M1**	3.1a
	$(5t-31)^2=(3t^2-25t+8)^2 \Rightarrow ...$ $9t^4-150t^3+648t^2-90t-897=0$	**M1** **A1**	1.1b 3.4
	Solves $9t^4-150t^3+648t^2-90t-897=0 \Rightarrow t=...$ $t=5\pm 2\sqrt{3}, \dfrac{23}{3}, \{-1\}$	**M1** **A1**	2.1 3.4
	Selects "insides" $(-1<)\alpha<t<\beta, \gamma<\mathrm{t}<\delta$ where $\alpha<\beta<\gamma<\delta$	**M1**	2.2a
	$(-1<)0 \leqslant t<5-2\sqrt{3}$ or $\dfrac{23}{3}<t<5+2\sqrt{3}$ condone any variable	**A1**	1.1b
	Both regions, $0 \leqslant t<5-2\sqrt{3}$ and $\dfrac{23}{3}<t<5+2\sqrt{3}$ must be using t	**A1**	2.3
		(8)	
(b)	Time that B is closer to O than particle A is $5+2\sqrt{3}-{}^{23}\!/_{3}+5-2\sqrt{3}={}^{7}\!/_{3}$ seconds.	**M1**	3.4
	This is considerably less than 4 seconds so the model does not seem appropriate.	**A1ft**	3.5a
		(2)	
			(10 marks)

Notes:

(a)
M1: Sets problem up as an inequalities problem, and forms complete strategy to solve – must see attempt at all critical values and some attempt to form at least one range from them. May be scored if algebra not used. This mark is for showing an overall awareness of the problem.

M1: Attempts to find C.V.'s for the "positives". Any valid method using algebra. Must see an attempt to find a 3TQ (oe), but allow answers from calculator once a 3TQ =0 is seen.

A1: Correct C.V.'s, both required.

M1: Attempts to find the other C.V.'s (same conditions as above)

A1: Correct C.V.'s. Need not see the negative value stated as $t > 0$ is required. (If both given, they must be correct)

M1: Selects correct critical regions, shows the idea the "insides" are needed.
$(-1<)\alpha < t < \beta, \gamma < \text{t} < \delta$ where $\alpha < \beta < \gamma < \delta$ are their four critical values, possibly truncated at 0 as long as no more than 1 is negative.

A1: One correct interval. Allow with loose or strict inequalities. Allow this mark if $-1 < t < 5 - 2\sqrt{3}$ is given. Allow any variable for this mark.

A1: Fully correct solution. Must start at zero for the leftmost interval but accept < or „ here. Must be using *t*

(a) Alternative

M1: Sets problem up as an inequalities problem, and forms complete strategy to solve – must see attempt at all critical values and some attempt to form at least one ranges from them. May be scored if algebra not used. This mark is for showing an overall awareness of the problem.

M1: Attempts to find C.V.'s by squaring both sides and forming a quartic equation.

A1: Correct quartic equation

M1: Attempts to solve their quartic equation

A1: All 4 correct exact C.V.'s. Need not see the negative value stated as $t > 0$ is required.

M1: Selects correct critical regions, shows the idea the "insides" are needed.
$(-1<)\alpha < t < \beta, \gamma < \text{t} < \delta$ where $\alpha < \beta < \gamma < \delta$ are their four critical values, possibly truncated at 0 as long as no more than 1 is negative.

A1: One correct interval. Allow with loose or strict inequalities. Allow this mark if $-1 < t < 5 - 2\sqrt{3}$ is given. Allow any variable for this mark.

A1: Fully correct solution. Must start at zero for the leftmost interval but accept < or „ here. Must be using *t*

(b)

M1: Uses their result from (a) to determine how long particle *B* is closer to *O* than particle *A* is.

A1: Draws a suitable conclusion for their answer to (a) – if correct in (a) it is that the model is not very suitable. Must not have a negative time.

Question	Scheme	Marks	AOs
7(a)	Gradient of $PQ = \dfrac{18q-18p}{9q^2-9p^2} = \dfrac{2}{p+q}$ $\begin{array}{l}18p = 9p^2m + c\\ 18q = 9q^2m + c\end{array} \Rightarrow 18p - 18q = 9p^2m - 9q^2m$ $\Rightarrow m = \dfrac{2}{p+q}$	**B1**	2.2a
	Equation of l is $y - 18p = "\dfrac{2}{p+q}"(x - 9p^2)$ $18p = 9p^2\left(\dfrac{2}{p+q}\right) + c$ $\Rightarrow c = \ldots$	**M1**	1.1b
	Leading to $(p+q)y = 2(x + 9pq)$ *	**A1***	2.1
		(3)	
(b)	Complete method to find equation of both normals and attempts to solve simultaneously	**M1**	3.1a
	E.g. $2y\dfrac{\mathrm{d}y}{\mathrm{d}x} = 36 \Rightarrow m_T = \dfrac{36}{36p} \Rightarrow m_N = -p$	**B1**	1.1b
	Normal at P is $y - 18p = -p(x - 9p^2)$ or normal at Q is $y - 18q = -q(x - 9q^2)$ (oe)	**M1**	2.1
	Both normals correct $y - 18p = -p(x - 9p^2)$ or $y = -px + 9p^3 + 18p$ (o.e.) $y - 18q = -q(x - 9q^2)$ or $y = -qx + 9q^3 + 18q$ (o.e.)	**A1**	2.2a
	E.g. $18p - px + 9p^3 - 18q = -qx + 9q^3 \Rightarrow x = \ldots$	**M1**	1.1b
	Need to show that $\left(9p^3 - 9q^3 + 18p - 18q\right) = \left(9p^2 + q^2 + pq + 2\right)\left(p - q\right)$ or $p^3 - q^3 = \left(p^2 + pq + q^2\right)\left(p - q\right)$ Leading to $x_A = 9(p^2 + q^2 + pq + 2)$ *	**A1***	2.2a
	$y = -9p(p^2 + q^2 + pq + 2) + 9p^3 + 18p = -9p^2q - 9pq^2$ Leading to $y_A = -9pq(p+q)$ *	**A1***	2.2a
		(7)	
(c)	(12,0) on $l \Rightarrow pq = -\dfrac{4}{3}$ (oe)	**B1**	3.1a
	Hence $x_A = 9\left(p^2 + q^2 + \dfrac{2}{3}\right)$ and $y_A = 12(p+q)$	**M1**	1.1b
	$y^2 = 144(p^2 + q^2 + 2pq) = 144\left(\dfrac{x}{9} - \dfrac{2}{3} + 2\left(-\dfrac{4}{3}\right)\right)$	**M1**	3.1a
	$y^2 = 16\left(x - 30\right)$ or $y^2 = 16x - 480$	**A1**	1.1b
		(4)	

(14 marks)
Notes:
(a) **B1:** Deduces gradient is $\frac{2}{p+q}$. May be implied by correct simplification of equation if the unsimplified form is used to start with. **M1:** Correct method for the equation of the line (gradient need not be simplified/correct for this method, as long as it is clearly an attempt at the gradient). **A1*:** Completes to the correct equation with no errors seen.
(b) **M1:** A correct overall method – must find both normals and attempt to solve simultaneously. They do not need to reach $x =$ or $y =$ as long as they have eliminated one variable. **B1:** Correct gradient of normal found from any correct method or just stated. **M1:** A full correct method to find the equation of at least one of the normals with justification of the gradient shown. **A1:** Deduces equation of the second normal – so both correct. **M1:** Solves the two normal equations simultaneously leading to either $x = \ldots$ or $y = \ldots$ **A1*:** Need to show that $(9p^3 - 9q^3 + 18p - 18q) = (9p^2 + q^2 + pq + 2)(p - q)$ leading to correct x coordinate with no errors seen. This could by long division or factorising. **A1*:** Correct y coordinate with no errors seen.
(c) **B1:** Uses the condition on l to establish the relationship between p and q **M1:** Uses their relationship between p and q to simplify the expressions **M1:** Any complete method for relating x and y independently of p and q **A1:** $y^2 = 16(x-30)$ or $y^2 = 16x - 480$

Question	Scheme	Marks	AOs
8(a)	$\text{f}(x)=\dfrac{3}{13+6\times\dfrac{2t}{1+t^2}-5\times\dfrac{1-t^2}{1+t^2}}$	**M1**	1.1b
	$=\dfrac{3(1+t^2)}{13(1+t^2)+12t-5(1-t^2)}$	**M1**	1.1b
	$=\dfrac{3(1+t^2)}{18t^2+12t+8}\Rightarrow$ for example $\dfrac{3(1+t^2)}{2(9t^2+6t+1)+6}$ or $\dfrac{3(1+t^2)}{2\left[(3t+1)^2-1\right]+8}$ $\Rightarrow\dfrac{3(1+t^2)}{2(3t+1)^2+6}$ *	**A1***	2.1
		(3)	
(b)	$\text{f}(x)=\dfrac{3}{7}\Rightarrow\dfrac{3(1+t^2)}{2(3t+1)^2+6}=\dfrac{3}{7}\Rightarrow 21+21t^2=54t^2+36t+24$ $\Rightarrow 11t^2+12t+1=0$	**M1**	1.1b
	$\Rightarrow(11t+1)(t+1)=0\Rightarrow t=\ldots$	**M1**	1.1b
	$t=-1,\ t=-\dfrac{1}{11}$	**A1**	1.1b
	$\Rightarrow x=2\arctan\left(\text{"}\textit{their } t\text{"}\right)+2\pi$ for a negative t	**dM1**	3.1a
	$x=\dfrac{3\pi}{2}$ or awrt 4.71 and awrt $x=6.10$	**A1**	1.1b
		(5)	
(c)	$\displaystyle\int\text{f}(x)=\int\frac{3(1+t^2)}{2(3t+1)^2+6}\times\frac{2}{1+t^2}\,\text{d}t=\int\frac{3}{(3t+1)^2+3}\,\text{d}t$	**B1**	2.1
	$=K\arctan\left(M(3t+1)\right)$ | $u=(3t+1)\Rightarrow K\arctan\left(Mu\right)$	**M1**	1.1b
	$=\dfrac{1}{\sqrt{3}}\arctan\left(\dfrac{3t+1}{\sqrt{3}}\right)$ | $=\dfrac{1}{\sqrt{3}}\arctan\left(\dfrac{u}{\sqrt{3}}\right)$	**A1**	1.1b
	$\displaystyle\int_{\frac{\pi}{3}}^{\frac{4\pi}{3}}\text{f}(x)\,\text{d}x=\int_{\frac{\pi}{3}}^{\pi}\text{f}(x)\,\text{d}x+\int_{\pi}^{\frac{4\pi}{3}}\text{f}(x)\,\text{d}x$ $\displaystyle=\int_{\frac{\sqrt{3}}{3}}^{\infty}\ldots\,\text{d}t+\int_{-\infty}^{-\sqrt{3}}\ldots\,\text{d}t$ or $\displaystyle\int_{\sqrt{3}+1}^{\infty}\ldots\,\text{d}u+\int_{-\infty}^{1-3\sqrt{3}}\ldots\,\text{d}u$	**B1**	3.1a
	$=\dfrac{1}{\sqrt{3}}\arctan\left(\dfrac{3\left(-\sqrt{3}\right)+1}{\sqrt{3}}\right)-\dfrac{1}{\sqrt{3}}\arctan\left(\dfrac{3\left(\dfrac{\sqrt{3}}{3}\right)+1}{\sqrt{3}}\right)+\ldots$	**M1**	1.1b

	$=\frac{\sqrt{3}}{3}\left(\arctan\left(\frac{\sqrt{3}-9}{3}\right)-\arctan\left(\frac{\sqrt{3}+3}{3}\right)\right)+\ldots$	A1	1.1b
	$=\ldots+\lim_{t\to\infty}\frac{1}{\sqrt{3}}\arctan\left(\frac{3t+1}{\sqrt{3}}\right)-\lim_{t\to-\infty}\frac{1}{\sqrt{3}}\arctan\left(\frac{3t+1}{\sqrt{3}}\right)=\ldots+\frac{\pi}{2\sqrt{3}}-\left(-\frac{\pi}{2\sqrt{3}}\right)$	M1	3.1a
	$=\frac{\sqrt{3}}{3}\left(\arctan\left(\frac{\sqrt{3}-9}{3}\right)-\arctan\left(\frac{\sqrt{3}+3}{3}\right)+\pi\right)$	A1	2.1
		(8)	
			(16 marks)

Notes:

(a)

M1: Uses one correct substitution

M1: Both substitutions correct and attempts to multiply through numerator and denominator by $1+t^2$.

A1*: Completes to the correct expression with no errors seen. Must see an intermediate step simplifying the denominator – most likely one of the ones seen in the scheme.

(b)

M1: Equates the result in (a) to $\frac{3}{7}$ and simplifies to a 3TQ

M1: Solves their equation by any valid means.

A1: Correct values for t

dM1: Dependent on first method mark. Applies the correct process to find at least one value for x from a negative value for t. (If two positive values are found in error, this mark cannot be scored.)

A1: Both answers correct and no others in range.

(c)

B1: Applies the substitution including the use of $\mathrm{d}x=\frac{2}{1+t^2}\mathrm{d}t$

M1: Attempts the integration to achieve $K\arctan\left(M(1+3t)\right)$ or $K\arctan\left(Mu\right)$ if using a substitution of $u=\left(3t+1\right)$.

May use substitution $3t+1=\sqrt{3}\tan\theta \Rightarrow \frac{\mathrm{d}t}{\mathrm{d}\theta}=\frac{\sqrt{3}}{3}\sec^2\theta \Rightarrow \int\frac{\sqrt{3}}{3}\mathrm{d}\theta=\frac{\sqrt{3}}{3}\theta=\frac{\sqrt{3}}{3}\arctan\left(\frac{3t+1}{\sqrt{3}}\right)$

A1: Correct integral.

B1: Changes the limits and splits the integral around π

M1: Applies their limits '$1/\sqrt{3}$' and '$-\sqrt{3}$' to their integrand.

A1: Correct "arctan" expressions.

M1: Correct work to evaluate the $\pm\infty$ limits

A1: Fully correct solution.